Imperial College London

Lecture 14

Discrete Fourier Transform
(Lathi 8.4-8.5)

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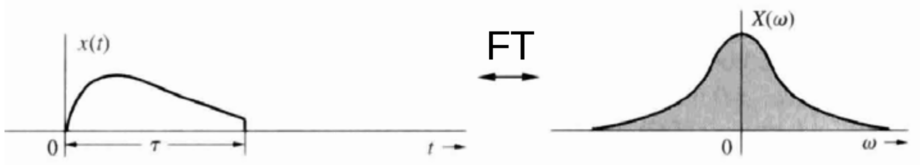
Spectral Sampling (1)

- As expected, time sampling has a dual: spectral sampling.
- Consider a time limited signal x(t) with a spectrum X(ω).

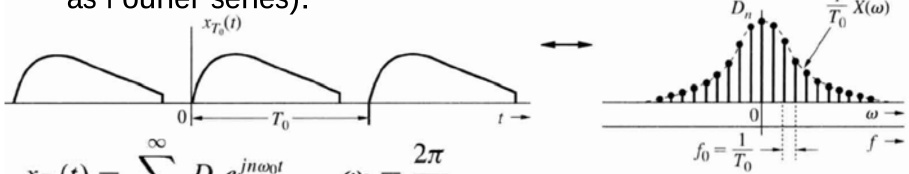


$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt = \int_0^{\tau} x(t)e^{-j\omega t}dt$$

L8.4 p796

Spectral Sampling (2)

- If we now CONSTRUCT a periodic signal $x_{T_0}(t)$, we will expect the spectrum of this signal to be discrete (expressed as Fourier series).



$$x_{T_0}(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t} \qquad \omega_0 = \frac{2\pi}{T_0}$$

where $D_n = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-jn\omega_0 t}dt = \frac{1}{T_0}\int_0^{\tau} x(t)e^{-jn\omega_0 t}dt$

therefore $$D_n = \frac{1}{T_0}X(n\omega_0)$$

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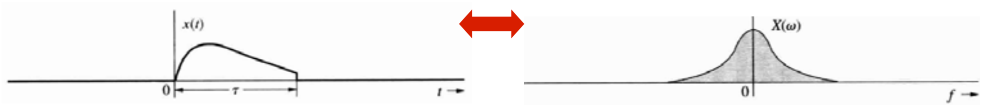
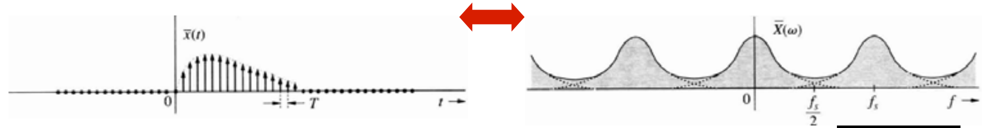
The Discrete Fourier Transform (DFT) (1)

- Fourier transform is computed (on computers) using discrete techniques.
- Such numerical computation of the Fourier transform is known as Discrete Fourier Transform (DFT).
- Begin with time-limited signal x(t), we want to compute its Fourier Transform X(ω).



- We know the effect of sampling in time domain:



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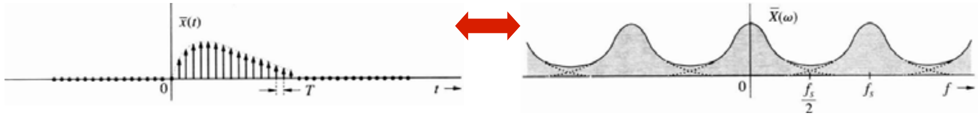
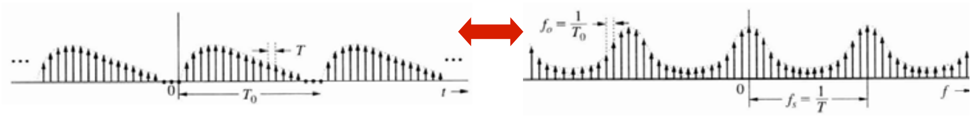
The Discrete Fourier Transform (DFT) (2)



- Now construct the sampled version of x(t) as repeated copies. The effect (from slides 2-3) is sampling the spectrum.



Number of time samples in T_0

$$N_0 = \frac{T_0}{T}$$

Number of frequency samples in f_s

$$N_0' = \frac{f_s}{f_0}$$

$$N_0 = \frac{T_0}{T} = \frac{1/f_0}{1/f_s} = \frac{f_s}{f_0} = N_0'$$

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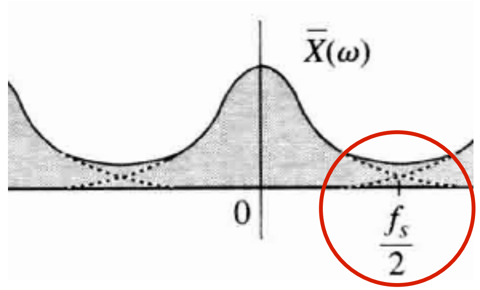
Aliasing and Leakage Effects

- Since X(ω) is not bandlimited, we will get some aliasing effect:



- Furthermore, if x(t) is not time limited, we need to truncate x (t) with a window function. This leads to leakage effect (as discussed in last lecture).

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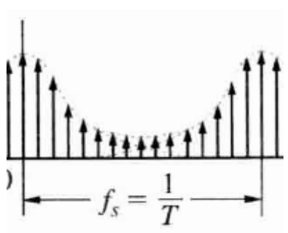
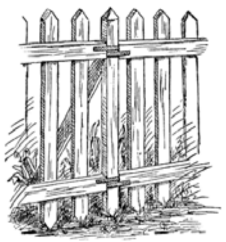
Picket Fence Effect

- Numerical computation method yields uniform sampling values of X(ω).
- Information between samples in spectrum is missing – picket fence effect:
- Can improve spectral resolution by increasing T.





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Formal definition of DFT

- If x(nT) and X(rω$_0$) are the nth and rth samples of x(t) and X(ω) respectively, then we define:

$$x_n = T x(nT) = \frac{T_0}{N_0} x(nT) \quad \text{and} \quad X_r = X(r\omega_0)$$

where $\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$

- Then

Forward DFT

$$X_r = \sum_{n=0}^{N_0-1} x_n e^{-jr\Omega_0 n}$$

$$\Omega_0 = \omega_0 T = \frac{2\pi}{N_0}$$

Inverse DFT

$$x_n = \frac{1}{N_0} \sum_{r=0}^{N_0-1} X_r e^{jr\Omega_0 n}$$

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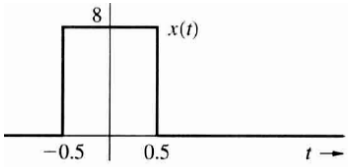
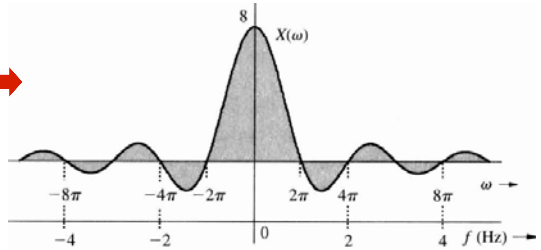
Example (1)

- Use DFT to compute the Fourier Transform of 8*rect(t).





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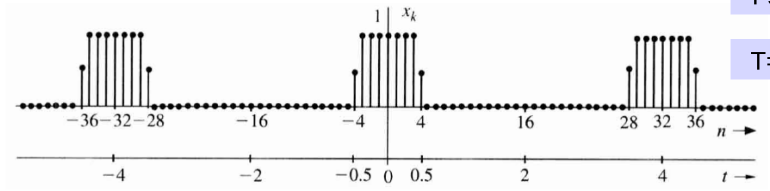
Example (2)

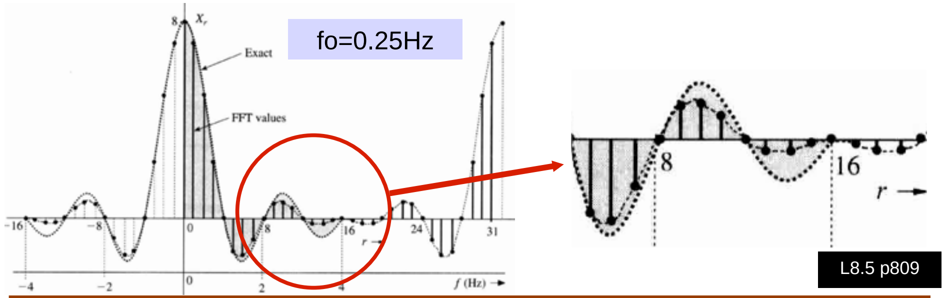
- After sampling and repetition:

Fs=8Hz

T=4 sec



fo=0.25Hz



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